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TWO-ZONE MODEL OF HEAT TRANSFER IN A FURNACE

S. P. Detkov and O. A. Bryukhovskikh UDC 536.3

A model of heat transfer in a furnace is proposed. The model is in the form of a circular channel separated lengthwise into sections. Each section is represented by two coaxial zones. A single section models a one-chamber furnace.

All of the engineering methods used to calculate heat transfer deal with single volumes. The volume is usually characterized by an effective temperature. The attempts made to calculate this temperature have generally produced results which are unreliable and, in truth, could not help but be unreliable. The formula derived by A. M. Gurvich [1] for use in the normative method does away with the notion of effective temperature but is also of limited application [2]. In the new single-volume method developed in [3, 4], the nonisothermality of the medium is accounted for through different values for the heat flux on the surface of the volume. However, these differences necessitate the solution of an internal problem. Simultaneous solutions of internal and external problems have been obtained by zone methods, but they are complicated and fail to meet several requirements. Below, we study a relatively simple two-zone model.

1. Physical Scheme. If the combustion chamber is long (such as in a rotary or continuous furnace), it is represented in the form of a channel with a flowing medium. The channel is divided into sections, but until now the practice has been to calculate heat transfer in each section in accordance with a single-volume model. The estimate obtained in [2] showed that heat flux is considerably overstated and that there is a corresponding underestimation of the temperature of the outgoing gases. In the present investigation, the volume is subdivided into two coaxial zones. In a reverberatory furnace, it is possible to distinguish a flame core or central layer (CL) and a more or less conservative layer (NL) which separates the core from the heating surface.

In our model, planarity of the flame is not required. In large furnaces, this provision leads only to a local increase in heat transfer or necessitates the installation of multiple burners that complicate the design of the furnace. Uniformity of the heat fluxes is usually the foremost criterion in determining the quality of heating, so planarity is undesirable.

2. System of Bodies. A circular channel or variable cross section is divided into sections of constant cross section. Two of them, i, and i-1, are shown in Fig. 1. The volumes of the medium are represented by two coaxial layers, CL and NL. The surface of the channel receives heat uniformly. The fields of all pertinent variables are axisymmetric. The equations describe heat transfer in the i-th section with allowance for the flow of the medium entering from section i − 1. The parameters are recalculated at the boundary. If there is just one section, then this section models a one-chamber furnace.

3. General Conditions of the Heat-Transfer Problem. In the i-th section of the channel:

1) the temperature field does not change along the axis;

2) the central layer (CL) is isothermal; its temperature and the field in the NL are found in the course of solivng the problem;

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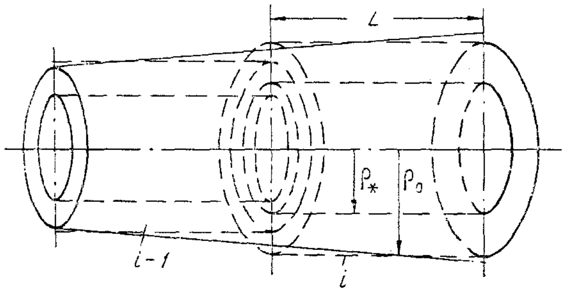

Fig. 1. Two channel sections, i-1 and i, each of which is represented by two volumetric zones and one surface zone. All of the zones are coaxial.

3) all of the fuel is burned in the CL; the NL contains no primary energy sources;

4) the velocity profile of the medium is determined by a law with an exponent of 1/7;

5) the conservativeness of the NL is maintained both for convective heat flow and for radiant flow;

6) the core is optically dense and its transmissivity is negligible; the model applies to large furnaces;

7) the core is replaced by an equivalently-radiating surface with the same temperature (this was show in [5] for a plane layer); in the NL, heat transfer occurs between coaxial, isotropic gray surfaces with the solution taken from [6].

4. Boundary of the Layers. The thickness of the NL to a large part determines the heat transfer that takes place. Thus, its value cannot be selected arbitrarily. The core has a fairly distinct boundary [7] when the field of energy sources is uniform. This boundary will be even more distinct in the case of diffusional combustion of fuel at the boundary with a jet of oxidizing agent. However, no suitable algorithm has been developed for calculating the flame front. We took an approach in which the boundary is determined during the solution of the problem on the basis of infiltration, leakage, and internal flow of the medium with allowance for convective heat transfer.

On the coaxial boundaries of the NL, convection occurs partially without resulting mass transfer. As usual, we assume the following on the surface of the channel:

$$q_c^* = \alpha_c(T_\Delta - T_0).$$

Turbulent pulsations of the medium participate in the heat-transfer process at the boundary between the layers (interface). Heat transfer is significantly influenced by the resulting flows of the medium: flow from the CL to the NL; flow from the NL to the environment through discontinuities in the channel (leaks); infiltration of air; flow from the previous section of the channel. The model accounts for all of these effects. At the same time, the model does not provide a direct connection between the movement of the medium and the fuel combustion system. The combustion system of course affects the size of the core of the flame.

Since the radiant heat transfer that takes place is an order of magnitude greater than the convective heat transfer, we will assume that the temperature field in the NL is formed by the mechanism of radiation and is independent of the convective flow.

5. Hydrodynamics of the Medium. If the model is a one-chamber model, then the mean velocity of the medium in the outlet section will be equal to

$$\overline{w} = Bv/(\pi\rho_0^2).$$

The veoocity profile has the form

$$w/w_0 = (1 - \rho/\rho_0)^{1/7},$$

where ρ is the radius reckoned from the axis of the channel, m; w_0 is the mean velocity on the axis, m/sec. Averaging yields the equality $\overline{w} = 0.817w_0$, so that $w/\overline{w} = (1 - \rho/\rho_0)^{1/7}$.

We will assume that the rate of flow of the medium $w\gamma$, kg/(m²·sec), is independent of temperature. Thus, the fractional flow rate in the NL, represented by δ, can be determined under normal conditions. We find that

$$\delta = z_*/0{,}408\,(3), \quad (1)$$

where $z_* = 7/8u_*^{8/7} - 7/15u_*^{15/7}$; $u_* = 1 - y_*$; $y_* = \rho_*/\rho_0$; y_* and u_* are the relative thickness of the CL and NL.

We write the dimensionless mean-mass temperatures: in the core of the flame, $\theta_* = T_*/T_a$ = const; in the NL

$$\theta_\Delta = \int_0^{z_*} \theta dz/z_*. \tag{2}$$

Over the complete cross section of the channel, the temperature of the outgoing gases

$$\theta_y = \theta_* (1 - \delta) + \theta_\Delta \delta. \tag{3}$$

6. Heat-Transfer Equations. The radiant flux in a system of gray bodies is equal to the following (on the surface F_0)

$$q_r' = a' (\theta_*^4 - \theta_0^4). \tag{4}$$

We take the following from (6):

$$a' = \frac{\rho_*}{\rho_0} \Big/ \left(\frac{1}{A_*} + \xi_* + \frac{\rho_* R_0}{\rho_0 A_0}\right), \tag{5}$$

where $\xi_* = 0.75k\rho_*\ln(\rho_0/\rho_*)$, $k = \alpha + \beta$.

The albedo of the elements within the volume influences heat transfer indirectly, through A_*. Since $A_* < 1$, the reflectivity of the surface of the CL is nontrivial, $R_* = 1 - A_* > 0$.

We take into account the selective properties only for the medium in the NL. We take the spectrum of the layer to be extreme and anti-gray [3, 4]. In this case, the heat flux is determined again from Eq. (4), but with the effective transmissivity

$$a'' = D \frac{\rho_*}{\rho_0} \Big/ \left(\frac{1}{A_*} + \frac{\rho_* R_0}{\rho_0 A_0}\right), \tag{6}$$

where $D = 1/(1 + \xi_*)$ is the transmissivity of the NL with a single passage of rays through the layer; the value of ξ_* is the same as for a graybody.

The temperature field has the following values:

in the core (CL), θ_* = const;
at the interface in the gray medium (discontinuity)

$$\theta_*^4 - \theta_{*0}^4 = q_r' \frac{\rho_0}{\rho_*} \left(\frac{1}{2} + \frac{R_*}{A_*}\right);$$

within the NL

$$\theta^4 - \theta_{00}^4 = q_r' \frac{\rho_0}{\rho} \left[\xi + \frac{1}{2} \left(1 - \frac{\rho}{\rho_0}\right)\right]; \tag{7}$$

at the boundary between the NL and the surface of the channel (discontinuity)

$$\theta_{00}^4 - \theta_0^4 = q_r' \left(\frac{1}{2} + \frac{R_0}{A_0}\right).$$

In (7), $\xi = 0.75k\rho \ln(\rho_0/\rho)$; if we take $\rho = \rho_*$ and we add the equations, we obtain a form of Eq. (4) which includes Eq. (5).

With an anti-gray spectrum for the NL, there are no temperature discontinuities at its boundaries:

$$\frac{\theta^4 - \theta_0^4}{\theta_*^4 - \theta_0^4} = \frac{\ln(\rho_0/\rho)}{\ln(\rho_0/\rho_*)}.$$

It can be seen that θ is independent of q_r'' in this case. The formulas presented above are needed to calculate θ_Δ from Eq. (2). It should be noted that, with allowance for a' and a'' from (5) and (6), different values are obtained for θ_*' and θ_*''.

As in [2], the Botlsmann number has the following form: $\mathrm{Bo} = Bv\bar{c}/(F_0\sigma T_a^3)$. Assigning the values of ρ_0 and T_a and the thermal load g (in W/m^2) for the entire volume, we find that

$$\mathrm{Bo} = g\rho_0/(2\sigma T_a^4).$$

Convection on the channel surface is calculated from the formula $q_c = \alpha_c(\theta_\Delta - \theta_0)/\sigma T_a^3$. Thus,

$$q_c/\mathrm{Bo} = 2\alpha_c T_a(\theta_\Delta - \theta_0)/(g\rho_0).$$

7. Balance Equations. The Boltzmann number $B_o = V\bar{c}/F_0\sigma T_a^3)$, where $V = Bv$ is the rate of flow of the combustion products in nm^3/sec, can also characterize other flow rates: the rate of infiltration V_f, the rate of leakage $V_{\ell e}$, the rates of flow on the boundary sections V_{i-1} and V_i, exchange at the boundary of the layers due to turbulent eddies and puslations V_*. In these cases, the following indices are obtained, respectively: Bo_f, $Bo_{\ell e}$, Bo_{i-1}, Bo_i, and Bo_*. We multiply the flow-rate equation by $\bar{c}/F_0\sigma T_a^3)$, assuming $\bar{c}$ to be the same in all of the terms. Infiltration and leakage will alternate in furnaces with a pressure close to atmospheric. When averaging is done over a sufficiently long period of time, the terms corresponding to both phenomena will be present in the equation. The equation (inlet at the left) for the CL takes the form

$$\mathrm{Bo}_{i-1}(1-\delta)+\mathrm{Bo}+\mathrm{Bo}_f(1-\delta)+\mathrm{Bo}_* = \mathrm{Bo}_i(1-\delta)+\mathrm{Bo}\,\delta + \\ + \mathrm{Bo}_{\ell e}1-\delta)+\mathrm{Bo}_*, \tag{8}$$

where $Bo_i = Bo_{i-1} + Bo + Bo_f - Bo_{\ell e}$.

Although all of the fuel burns in the CL, the velocity profile is assumed to be unchanging. We find that there is a forced flow of the medium from the CL to the NL in the amount $Bo\delta + Bo_{\ell e}(1-\delta)$. The flow in the opposite direction from the NL to the CL is $Bo_f(1-\delta)$.

The material balance equation for the NL:

$$\mathrm{Bo}_{i-1}\,\delta+\mathrm{Bo}\,\delta+\mathrm{Bo}_f+\mathrm{Bo}_{\ell e}(1-\delta)+\mathrm{Bo}_* = \mathrm{Bo}_i\,\delta + \\ + \mathrm{Bo}_f(1-\delta)+\mathrm{Bo}_{\ell e}+\mathrm{Bo}_*. \tag{9}$$

Termwise multiplication of (8) by the temperature of the flows yields the convective terms of the heat balance equation (for Bo at the inlet, $\theta_a = 1$). The difference between the terms is equal to the heat flux

$$q_r = \mathrm{Bo}(1-\theta_*)-\mathrm{Bo}_{i-1}(1-\delta)(\theta_{*i}-\theta_{*i-1})-[\mathrm{Bo}_f(1-\delta)+\mathrm{Bo}_*](\theta_*-\theta_\Delta). \tag{10}$$

Leakage of the medium does not directly affect q_r. Accordingly, in Eq. (9) the heat balance of the NL determines the convective flow

$$q_c = \mathrm{Bo}\,\delta(\theta_*-\theta_\Delta)-\mathrm{Bo}_{i-1}\,\delta(\theta_{\Delta i}-\theta_{\Delta i-1})-\mathrm{Bo}_f(\theta_\Delta-\theta_f)+\mathrm{Bo}_*(\theta_*-\theta_\Delta) + \\ + \mathrm{Bo}_{\ell e}(1-\delta)(\theta_*-\theta_\Delta). \tag{11}$$

It can be seen that leakage intensifies convection, but in this case there is a reduction in the flow of energy to subsequent sections due to Bo_i. The radiant flux passing through the NL leaves Eq. (11) unchanged. The condition for conservativeness of the NL is also satisfied when no fuel combustion takes place. Equation (11) should be used to determine δ. Then Eq. (1) is used to find ρ_*/ρ_0.

The addition of (10) and (11) gives the heat balance for a section of the channel in the form

$$q = q_r + q_c = \mathrm{Bo}(1-\theta_g)+\mathrm{Bo}_{i-1}(\theta_{g i-1}-\theta_{gi})-\mathrm{Bo}_f(\theta_g-\theta_f) + \\ + \mathrm{Bo}_{\ell e}(1-\delta)(\theta_*-\theta_\Delta). \tag{12}$$

Here, we considered Eq. (3).

8. Special Case of a One-Chamber Furnace. As an example, we will examine a single section of a channel with $V_{i-1} = 0$. At the same time, we put $V_{\ell e} = 0$. We thus obtain:

$$q_r = \mathrm{Bo}(1-\theta_*)-[\mathrm{Bo}_f(1-\delta)+\mathrm{Bo}_*](\theta_*-\theta_\Delta), \tag{13}$$

$$q_c = \mathrm{Bo}\,\delta(\theta_*-\theta_\Delta)-\mathrm{Bo}_f(\theta_\Delta-\theta_f)+\mathrm{Bo}_*(\theta_*-\theta_\Delta). \tag{14}$$

The following computing method will be used. In Eq. (13) we substitute q_r' found from (4) and (5) or q_r'' from (4) and (6). We calculate the pyrometric temperatures of the core of the flame θ_*' and θ_*''. the quantities δ, θ_Δ, and ρ_*/ρ_0 are assigned in the zeroth approxi-

TABLE 1. Effect of the Temperature of the Heat-Receiving Surface on Heat Transfer with α_c = 5 W/(m²·K); k = 2 m⁻¹; Bo_f/Bo = 0.1; Bo_*/Bo = 0.05

θ_0	ρ_*/ρ_0	δ	θ_*	θ_Δ	θ_g	q_r	q_c
			Gray NL				
0,1	0,73	0,41	0,79	0,66	0,74	0,117	0,004
0,3	0,73	0,41	0,79	0,67	0,74	0,116	0,003
0,5	0,69	0,48	0,81	0,70	0,76	0,103	0,002
			Anti-gray NL				
0,1	0,81	0,30	0,78	0,62	0,73	0,119	0,004
0,3	0,80	0,31	0,78	0,63	0,74	0,118	0,003
0,5	0,74	0,39	0,81	0,69	0,76	0,102	0,002

TABLE 2. Effect of Air Infiltration on Heat Transfer with k = 2 m⁻¹, Bo_*/Bo = 0.1

Bo_f/Bo	q_r	q_c	ρ_*/ρ_0	θ_g
0,05	0,136	0,003	0,86	0,74
0,10	0,119	0,003	0,76	0,74
0,15	0,103	0,003	0,66	0,73

TABLE 3. Effect of Internal Agitation of the Medium on Heat Transfer with k = 2 m⁻¹, Bo_f/Bo = 0.1

Bo_*/Bo	q_r	q_c	ρ_*/ρ_0	θ_g
0	0,114	0,003	0,70	0,74
0,05	0,117	0,003	0,73	0,74
0,10	0,119	0,003	0,76	0,74
0,15	0,121	0,003	0,78	0,73
0,20	0,121	0,003	0,80	0,73

mation and subsequently refined. The value of θ_Δ is found from (2), while θ_g is found from (3). The value of δ is calculated from (14):

$$\delta=\frac{q_c+Bo_f(\theta_\Delta-\theta_f)}{Bo(\theta_*-\theta_\Delta)}-\frac{Bo_*}{Bo}. \tag{15}$$

The cycle of computations is repeated until the specified accuracy is obtained. The addition of (13) and (14) yields the complete balance equation

$$q=Bo(1-\theta_g)-Bo_f(\theta_g-\theta_f). \tag{16}$$

This equation is not used in the calculations. Instead, it is employed to check the accuracy of the solution.

9. Numerical Calculation. The model was constructed for large furnaces. We take ρ_0 = 3 m (channel diameter 6 m). With an attenuation factor k = 2-3 m⁻¹, the central layer has a negligibly low transmissivity and can be replaced by an equivalent surface. The deviation of its emissivity from unity, A_* = 0.95, indicates that energy is dissipated. direct calculations of A_* by the methods described in [8] have been omitted here. The value A_0 = 0.70 characterizes the contaminated surface of the channel. In accordance with the type of fuel used, we took T_a = 2277 K. The thermal load was given (g = 6·10⁶ W/m³) for the entire volume of the furnace, with θ_f = 0.15. The fouling of the heating surface is reflected in the temperature θ_0. Its value was assigned from the range 0.1-0.5. We assigned the infiltration Bof/Bo = 0-0.2, while turbulent exchange with the medium at the interface Bo_*/Bo = 0-0.2 and the convective heat-transfer coefficient α_c = 5-10 W/(m²·K). A value of 0.5905 was obtained for Bo and did not change in the course of the computations.

A fivefold increase in the temperature θ_0 had little effect on the heat flux (see Table 1), since the difference $\theta_*^4 - \theta_0^4$ decreased by only 2%. The thickness of the NL increased appreciably and became the main reason for the reduction in heat transfer. The effect of the spectrum on the heat flux turned out to be slight in our model, partly because the

CL remained gray and the value $A_* = 0.95$ was assumed to have been constant. Although the value of δ in the anti-gray spectrum of the NL decreased, the thermal resistance of the NL remained nearly constant.

The subsequent analysis was performed with a gray spectrum with $\theta_0 = 0.3$ and $\alpha_c = 5\ W/(m^2 \cdot K)$. Air infiltration had a significant effect (Table 2): it increased the thickness of the NL and, accordingly, reduced heat transfer with almost no change in the temperature of the outgoing gases. It is evident from Table 3 that heat transfer intensifies due to internal agitation of the medium. Heat flux increases somwehat with a reduction in the thickness of the NL.

An increase in the attenuation factor from 1 to 3 m^{-1} in the calculation affected only the thermal resistance of the NL. The thickness of the NL decreased, but heat flux also still decreased substantially, while θ_g increased. An increase in α_c was accompanied by an increase in the flux q_c. However, in accordance with (15), the thickness of the NL increased and the total heat flux even decreased, other conditions remaining the same.

Considering its specific features, the above model adequately describes actual conditions.

NOTATION

A, emissivity of the surface; $R = 1 - A$; F_0, lateral heat-receiving surface of section i, m^2; ρ_* and ρ_0, radii of the CL and the channel, m; L, section length, m; T_*, temperature of the CL (the core of the flame), K; T_0, T_Δ, T_g, temperature of the surface F_0, the NL, and the outgoing gases, K; B, rate of fuel consumption, kg/sec; v, specific volume of combustion products with allowance for excess air, $J/(nm^3 \cdot K)$; T_α, adiabatic fuel-combustion temperature, K; q_r^* and q_c^*, radiant and convective fluxes, W/m^2; α and β, absorption and scattering coefficients, m^{-1}; k, attenuation factor; δ, fractional flow of the medium in the NL; $1 - \delta)$ same in the CL; γ, density of medium, kg/m^3; dimensionless quantities; $Bo = \sqrt{c}/(F_0 \sigma T_\alpha^3)$; $\theta = T/T_\alpha$; $\theta_* = T_*/T_\alpha$; $\theta_0 = T_0/T_\alpha$; $\theta_y = T_y/T_\alpha$; $q_r = q_r^*/\sigma T_\alpha^4$; $q_c = q_c^*/\sigma T_\alpha^4$. Indices: f, ℓe, $*$, Δ, 0, infiltration, leakage of medium, CL, NL, surface F_0.

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